Abstracts for the Workshop « Asymptotic Analysis in General Relativity »

Lars Andersson : TBA

Alain Bachelot : Waves in the Anti-de Sitter space-time AdS^5.

Abstract : In this talk we address some issues concerning the wave propagation in the 4D+1 anti-de Sitter space-time : the role of the conformal boundary, the representation of the fields in term of Kaluza-Klein tower, the existence of new dynamics associated with a family of novel boundary conditions, the linear stability of a De Sitter brane.

Thomas Bäckdahl : Symmetry operators, conserved currents and energy momentum tensors.

Abstract : Conserved quantities, for example energy and momentum, play a fundamental role in the analysis of dynamics of particles and fields. For field equations, one manifestation of conserved quantities in a broad sense is the existence of symmetry operators, i.e. linear differential operators which take solutions to solutions. A well-known example of a symmetry operator for the scalar wave equation is provided by the Lie derivative along a Killing vector field. It is important to note that other kinds of objects can generate symmetry operators. For waves in the Kerr spacetime there is a symmetry operator associated with Carter's constant. This symmetry, which is "hidden" in the sense that it arises from a Killing spinor, satisfying a generalization of the Killing vector equation, rather than a Killing vector, was an essential ingredient in a proof of decay of scalar waves on the Kerr background by Andersson and Blue.
In this talk we will consider what conditions on a spacetime are necessary for existence of symmetry operators for the conformal wave equation, the Dirac-Weyl equation, and the Maxwell equation, i.e. for massless test fields of spins 0, 1/2 and 1. We will investigate how the conditions for the symmetry operators for the different field equations are related, and how they are related to existence of conserved currents. Furthermore, these tools lead to the construction of a new energy momentum tensor for a Maxwell field on a Kerr background. This will provide a powerful tool for the study of decay of Maxwell fields on the Kerr spacetime.

Pieter Blue : Decay for fields outside black holes.

Abstract : I will discuss energy and Morawetz (or integrated local decay) estimates for fields outside black holes. These results build on results for the wave equation and use the Killing tensor, an unusual geometric object that exists in the Kerr spacetime.

Mihalis Dafermos : TBA

Claudio Dappiaggi : On the role of asymptotic structures in the construction of quantum states for free field theories on curved backgrounds.

Abstract : In the algebraic approach to quantum field theory on curved backgrounds, there exists a special class of quantum states for free fields, called of Hadamard form. These are of particular relevance since they yield finite quantum fluctuations of all observables and they can be used to implement interactions at a perturbative level. Although their existence is guaranteed on all globally hyperbolic spacetimes, for long time only few explicit examples were known. A way to bypass this problem exists on those manifolds which possess a null conformal boundary, such as all asymptotically flat spacetimes. In this talk we shall discuss this construction in particular for massless, conformally coupled scalar fields and for linearised gravity.

Semyon Dyatlov : Spectral gaps for normally hyperbolic trapping.

Abstract : Motivated by wave decay for Kerr and Kerr-de Sitter black holes, we study spectral gaps for codimension 2 normally hyperbolic trapped sets with smooth stable/unstable foliations. Using semiclassical defect measures, we recover the gap of Wunsch-Zworski and Nonnenmacher-Zworski in our case. Under the stronger assumptions of r-normal hyperbolicity and pinching, we discover further spectral gaps, meaning that resonances are stratified into bands by decay rates.

Christian Gérard : Construction of Hadamard states for Klein-Gordon fields.

Abstract : we will review a new construction of Hadamard states for quantized Klein-Gordon fields on curved spacetimes, relying on pseudo differential calculus on a Cauchy surface. We also present some work in progress where Hadamard states are constructed from traces of Klein-Gordon fields on a characteristic cone. (Joint work with Michal Wrochna).

Rod Gover : Geometric Compactification, Cartan holonomy, and asymptotics.

Abstract : Conformal compactification has long been recognised as an effective geometric framework for relating conformal geometry, and associated field theories ``at infinity'', to the asymptotic phenomena of an interior (pseudo-)-Riemannian geometry of one higher dimension. It provides an effective approach for analytic problems in GR, geometric scattering, conformal invariant theory, as well as the AdS/CFT correspondence of Physics. I will describe how the notion of conformal compactification can be linked to Cartan holonomy reduction. This leads to a conceptual way to define other notions of geometric compactification. The idea will be taken up, in particular, for the case of compactifying pseudo-Riemannian manifolds using projective geometry. A new characterisation of projectively compact metrics will be given, and some results on their asymptotics near the conformal infinity. This is joint work with Andreas Cap.

Jérémie Joudioux : Hertz potentials and the decay of higher spin fields.

Abstract : The study of the asymptotic behavior of higher spin fields has proven to be a key point in understanding the stability properties of the Einstein equations. Penrose derived in the 60s the asymptotic behavior of these higher spin fields from a representation by Hertz potentiels satisfying a wave equation and a decay Ansatz for the solutions of the wave equation. The purpose of this talk is to perform the construction by Penrose in the context of the Cauchy problem on Minkowski space-time for Maxwell fields and linearized gravity. Considering a Cauchy problem for Maxwell fields and linearized gravity with data in weighted Sobolev spaces, a Hertz potential is build from a generalization of the de Rham complex to arbitrary spin. The asymptotic behavior of these higher spin fields is then derived from the asymptotic behavior of the solutions of the wave equation.

Niky Kamran : An Exact Expression for Photon Polarization in Kerr geometry.

Abstract : We analyze the transformation of the polarization of a photon propagating along an arbitrary null geodesic in Kerr geometry. The existing methods to calculate the rotation of the photon polarization (Faraday rotation) depend on choices of coordinate systems, are algebraically complex and yield results only in the weak-field limit. We give a closed-form expression for a parallel propagated frame along an arbitrary null geodesic using Killing-Yano theory, and thereby solve the problem of parallel transport of the polarization vector in an intrinsic, geometrically-motivated fashion. The symmetries of Kerr geometry are utilized to obtain a remarkably compact expression for the geometrically induced phase of the photon's polarization. We show that this phase vanishes on the equatorial plane and the axis of symmetry. This is joint work with Anusar Farooqui and Prakash Panangaden (McGill), arXiv:1306.6292, to appear in Advances in Theoretical and Mathematical Physics.

Sergiu Klainerman : TBA

Philippe G. LeFloch : Weakly regular spacetimes with T2 symmetry.

Abstract : I will discuss the initial value problem for the Einstein equations and present results concerning the existence and asymptotic behavior of spacetimes, when the initial data set is assumed to be T2 symmetric and satisfies weak regularity conditions so that the spacetimes may exhibit impulsive gravitational waves and shock waves. This lecture is based on papers written over the period 2004—2014 and available at philippelefloch.org.

Lionel Mason : Perturbative formulae for scattering of gravitational waves.

Abstract : The Christodoulou-Klainerman proof of existence of asymptotically simple space-times shows that it is reasonable to consider the scattering of characteristic data for the Einstein field equations from past null infinity to that on future null infinity in a neighbourhood of Minkowski space. In this talk I present new explicit perturbative formulae for this scattering for general data to arbitrary order. Unlike previous such formulae, these new formulae are not chiral, and naturally respect the real structures and may therefore be more amenable to analysis. This is based on joint work with Yvonne Geyer and Arthur Lipstein and with David Skinner.

Valter Moretti : On the so-called "tunnelling interpretation" of black-hole radiation form the algebraic QFT viewpoint.

Abstract : "Tunnelling processes" through black hole horizons have been investigated in the framework of WKB theory, discovering an interesting interplay with Hawking radiation. We instead [1] adopt the point of view of algebraic QFT in curved spacetime. Use a suitable scaling limit towards a Killing horizon we obtain the leading order of the correlation function relevant for the tunnelling. The computation is done for a certain large class of reference quantum states for scalar fields, including Hadamard states. In the limit of sharp localization on opposite sides of the horizon, the quantum correlation functions appear to have thermal nature with characteristic temperature given by Hawking's one. The approach is valid for stationary charged rotating non-extremal black hole and also covers the case of a Killing horizon which just temporarily exists in some finite region. These results provide strong support to the idea that the Hawking radiation is actually related to phenomena associated to local Killing horizons. Finally, [2] the theory is applied to the model of a ϕ^3 self-interaction in Rindler spacetime, and renormalization counterterms are computed, obtaining that Hawking radiation perturbatively survives the introduction of the interaction.
[1] V.M., N.Pinamonti, Commun.Math.Phys. 309,295–311(2012)
[2] G.Collini, V.M., N.Pinamonti, Lett.Math.Phys. 104,217–232(2014)

Jacques Smulevici : Future Dynamics of T2 symmetric polarized space times.

Abstract : Joint Work with Philippe G. LeFloch (Paris 6). We consider vacuum spacetimes with two spatial Killing vectors and with initial data prescribed on T^3. The main results that we will present concern the future asymptotic behaviour of the so-called polarized solutions. Under a smallness assumption, we derive a full set of asymptotics for these solutions. Within this symetry class, the Einstein equations reduce to a system of wave equations coupled to a system of ordinary differential equations. The main difficulty, not present in previous study of similar systems, is that, even in the limit of large times, the two systems do not directly decouple. We overcome this problem by the introduction of a new system of ordinary differential equations, whose unknown are renormalized variables with renormalization depending on the solution of the non-linear wave equations.

Alexander Strohmaier : On Gupta-Bleuler quantization in globally hyperbolic spacetimes with non-compact Cauchy surfaces.

Abstract : I will explain how one can formulate and formalize the Gupta-Bleuler framework for the Quantization of the electromagnetic

field in an algebraic manner so that it works on globally hyperbolic space-times. I will then discuss a construction of physical representations that works without the "spectral gap assumption" in the case of absense of zero energy resonances. These can be excluded by topologocial restrictions at infinity. This is based on joint work with Felix Finster

Jérémie Szeftel : The resolution of the bounded L2 curvature conjecture in general relativity.

Abstract : In order to control locally a space-time which satisfies the Einstein equations, what are the minimal assumptions one should make on its curvature tensor? The bounded L2 curvature conjecture roughly asserts that one should only need L2 bounds of the curvature tensor on a given space-like hypersurface. This conjecture has its roots in the remarkable developments of the last twenty years centered around the issue of optimal well-posedness for nonlinear wave equations. In this context, a corresponding conjecture for nonlinear wave equations cannot hold, unless the nonlinearity has a very special nonlinear structure. I will present the proof of this conjecture, which sheds light on the specific null structure of the Einstein equations. This is joint work with Sergiu Klainerman and Igor Rodnianski.

Andras Vasy : Quasilinear waves and trapping: Kerr-de Sitter space.

Abstract : In this talk I will describe recent work with Peter Hintz on globally solving quasilinear wave equations in the presence of trapped rays, on Kerr-de Sitter space, and obtaining the asymptotic behavior of solutions. For the associated linear problem without trapping, one would consider a global, non-elliptic, Fredholm framework; in the presence of trapping the same framework is available for spaces of growing functions only. In order to solve the quasilinear problem we thus combine these frameworks with the normally hyperbolic trapping results of Dyatlov and a Nash-Moser iteration scheme.

Maciej Zworski : From redshift effect to classical dynamics: a microlocal proof of Smale's conjecture.

Abstract: Dynamical zeta functions of Selberg, Smale and Ruelle are analogous to the Riemann zeta function with the product over primes replaced by products over closed orbits of Anosov flows. In 1967 Smale conjectured that these zeta functions should be meromorphic but admitted "that a positive answer would be a little shocking". Nevertheless the continuation was proved in 2012 by Giulietti-Liverani-Pollicott. By combining the Faure-Sjöstrand approach to Anosov flows and Melrose's microlocal radial estimates, Dyatlov and I gave a simple proof of that conjecture. The same radial estimates were used by Vasy to provide a microlocal explanation of the redshift effect and propagation estimates for Kerr-de Sitter-like spaces.