Summer school « Asymptotic Analysis in General Relativity « : Abstracts of the mini-courses.

Lars Andersson (Albert Einstein / Max Planck Institut Golm) : Geometry and analysis in black hole spacetimes.

Abstract: Black holes play a central role in general relativity and astrophysics. The problem of proving the dynamical stability of the Kerr black hole spacetime, which is describes a rotating black hole in vacuum, is one of the most important open problems in general relativity. Following a brief introduction to the evolution problem for the Einstein equations, I will give some background on geometry of the Kerr spacetime. The analysis of fields on the exterior of the Kerr black hole serve as important model problems for the black hole stability problem. I will discuss some of the difficulties one encounters in analyzing waves in the Kerr exterior and how they can be overcome.
A fundamentally important aspect of geometry and analysis in the Kerr spacetime is the fact that it is algebraically special, of Petrov type D, and therefore admits a Killing spinor of valence 2. I will introduce the 2-spinor and related formalisms which can be used to see how this structure leads to the Carter constant and the Teukolsky system. If there is time, I will discuss in this context some new conservation laws for fields of non-zero spin.

Christian Gérard (Université d'Orsay) : Introduction to quantum field theory on curved spacetimes.

*Abstract :*The aim of these lectures is to give an introduction to quantum field theory on curved spacetimes, from the point of view of partial differential equations and microlocal analysis. I will concentrate on free fields and quasi-free states, and say very little on interacting fields or perturbative renormalization. I will start by describing the necessary algebraic background, namely CCR and CAR algebras, and the notion of quasi-free states , with their basic properties and characterizations. I will then introduce the notion of globally hyperbolic spacetimes, and its importance for classical field theory (advanced and retarded fundamental solutions, unique solvability of the Cauchy problem). Using these results I willexplain the algebraic quantization of the two main examples of quantum fields ona manifold, namely the Klein-Gordon (bosonic) and Dirac (fermionic) fields.In the second part of the lectures I will discuss the important notion of Hadamardstates , which are substitutes in curved spacetimes for the vacuum state in Minkowskispacetime. I will explain its original motivation, related to the definition of therenormalized stress-energy tensor in a quantum field theory. I will then describethe modern characterization of Hadamard states, by the wavefront set of their twopointfunctions, and prove the famous Radzikowski theorem , using the Duistermaat-Hörmander notion of distinguished parametrices . If time allows, I will also describe the quantization of gauge fields, using as example the Maxwell field.

Rod Gover (University of Auckland) : An introduction to conformal geometry and tractor calculus, with a view to applications in GR.

Abstract : After recalling some features (and the value of) the invariant ``Ricci calculus'' of pseudo-Riemannian geometry, we look at conformal rescaling from an elementary perspective. The idea of conformal covariance is visited and some covariant/invariant equations from physics are recovered in this framework. Motivated by the need to develop a more effective approach to such problems we are led into the idea of conformal geometry and a conformally invariant calculus; this ``tractor calculus'' is then developed explicitly. We will discuss how to calculate using this, and touch on applications to the construction of conformal invariants and conformally invariant differential operators. The second part of the course is concerned with the application of conformal geometry and tractor calculus for the treatment of conformal compactification and the geometry of conformal infinity. The link with Friedrich's conformal field equations will be made. As part of this part we also dedicate some time to the general problem of treating hypersurfaces in a conformal manifold, and in particular arrive at a conformal Gauss equation. Finally we show how these tools may be applied to treat aspects of the asymptotic analysis of boundary problems on conformally compact manifolds.

Jérémie Szeftel (Université Paris VI) : The resolution of the bounded L2 curvature conjecture in General Relativity.

Abstract : In order to control locally a space-time which satisfies the Einstein equations, what are the minimal assumptions one should make on its curvature tensor? The bounded L2 curvature conjecture roughly asserts that one should only need L2 bounds of the curvature tensor on a given space-like hypersurface. This conjecture has its roots in the remarkable developments of the last twenty years centered around the issue of optimal well-posedness for nonlinear wave equations. In this context, a corresponding conjecture for nonlinear wave equations cannot hold, unless the nonlinearity has a very special nonlinear structure. I will present the proof of this conjecture, which sheds light on the specific null structure of the Einstein equations. This is joint work with Sergiu Klainerman and Igor Rodnianski. These lectures will start from scratch and require no specific background.

Andras Vasy (Stanford University) : Microlocal analysis and wave propagation.

Abstract : In these lectures I will explain the basics of microlocal analysis, emphasizing non-elliptic problems, such as wave propagation, both on manifolds without boundary, and on manifolds with boundary. In the latter case there is no `standard' algebra of differential, or pseudodifferential, operators; I will discuss two important frameworks: Melrose's totally characteristic, or b-, operators and scattering operators. Apart from the algebraic and mapping properties, I will discuss microlocal ellipticity, real principal type propagation, radial points and generalizations, as well as normally hyperbolic trapping. The applications discussed will include Fredholm frameworks (which are thus global even for non-elliptic problems!) for the Laplacian on asymptotically hyperbolic spaces and the wave operator on asymptotically de Sitter spaces, scattering theory for `scattering metrics' (such as the `large ends' of cones), wave propagation on asymptotically Minkowski spaces and generalizations (`Lorentzian scattering metrics') and on Kerr-de Sitter-type spaces. The lectures concentrate on linear PDE, but time permitting I will briefly discuss nonlinear versions. The lecture by the speaker in the final workshop will use these results to solve quasilinear wave equations globally, including describing the asymptotic behavior of solutions, on Kerr-de Sitter spaces.